PARTICLE-IN-A-BOX STUDY OF THE EVOLUTION OF AN ION-ACOUSTIC SHOCK WAVE

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In an analysis of a one-dimensional numerical model of a nonisothermal plasma it is shown that an ion-acoustic shock wave of subcritical amplitude separates a "soliton" from the shock front after the "reversing" stage. This process is accompanied by turbulent flow behind the front and by trapping of ions in potential wells. The numerical "particle-in-a-box" method is being used widely to study plasma phenomena. One field in which this method has been found fruitful is in the study of a nonisothermal plasma, characterized by an ion-acoustic wave branch.

The problem of the propagation of ion-acoustic waves can be solved either in the case of quasisteady-state wave propagation, in which case the unknowns depend only on the variable $\xi = x - ut$, or in the non-steady-state case for waves of small but finite amplitude. In both cases a hydrodynamic description of the plasma is used. It is assumed that, although the plasma is collisionless, its distribution is Maxwellian. After linearization and account of terms of up to third order, the Corteweg de Vrise equation, which describes wave propagation in a dispersive medium, is found. In the case of quasisteady-state wave propagation a relation between the maximum wave potential φ_{max} and the propagation velocity u can be found [2]:

$$u^2 = \frac{T}{2M} \frac{[\exp(e\varphi_{max}/T) - 1]^2}{\exp(e\varphi_{max}/T) - 1 - e\varphi_{max}/T}.$$

The hydrodynamic description is suitable for waves whose amplitudes and velocities do not exceed certain critical values [2].

The kinetic theory is not limited in this manner; it can be used to describe the evolution of waves having amplitudes and velocities above these critical values. Since it is extremely difficult to analytically solve the kinetic equation, the course of the process can be followed by using a computer to track particles. The calculation time can be reduced significantly without any distortion of the fundamental characteristics of the process by assuming a Boltzmann distribution $n_e = n_0 \exp(e\varphi/T)$ for the electron density.

We consider the one-dimensional problem. We represent the ionic component of plasma as charged planes perpendicular to the x axis and having a charge e and mass m_i equal to the charge and mass of the ion. The equation of motion of each such quasiparticle is

$$\frac{d^2x}{dt^2} = -\frac{e}{m_i}\nabla\varphi.$$

The self-consistent electric field is determined from the Poisson equation

$$\frac{d^2\varphi}{dx^2} = 4\pi e\,[n_0 \exp(e\varphi/T) - n_i].$$

In a real plasma there are a large number of particles per Debye length, so a real plasma cannot be simulated in this manner, since few particles are involved in a computer experiment. In this case the particles which leave one region in which the density is determined and enter another will cause important density perturbations.

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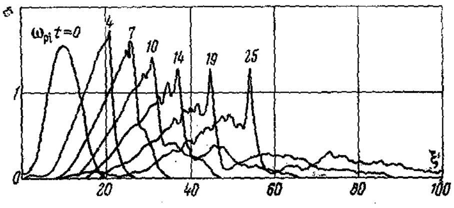

Fig. 1

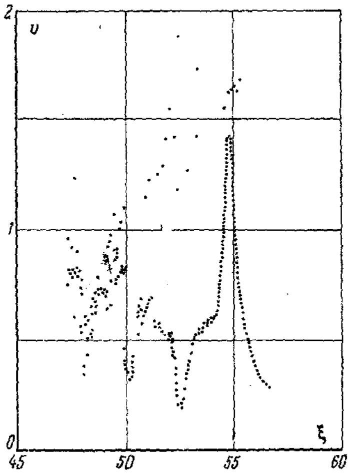

Fig. 2

To reduce the effect, we can "smear" this quasiparticle along the x coordinate. However, this model of smeared particles distorts certain of the dispersion properties. For convenience in the analysis we assume particles in which the density satisfies

$$\rho_i(x, x_j) = \frac{e}{a\sqrt{2\pi}} \exp\left[\frac{-(x-x_j)^2}{2a^2}\right],$$

where x_j is the center of the "cloud." The force acting on such a particle is defined as

$$F(x_j) = \frac{e}{a\sqrt{2\pi}} \int_{-\infty}^{\infty} E(x) \exp\left[\frac{-(x-x_j)^2}{2a^2}\right] dx.$$

When this force is taken into account, the dispersion relation becomes

$$1 = \frac{T}{m_i} \frac{\exp(-{}^1/_2 k^2 a^2)}{1+k^2D^2} \int_{-\infty}^{\infty} \frac{\partial f/\partial v}{v+\omega/k} dv.$$

This relation differs from the familiar dispersion relation in that it contains a factor $\exp(-k^2a^2/2)$, which would distort the high-frequency oscillations. These oscillations will be suppressed relatively severely at the larger half-widths a.

Since calculations based on the Poisson equation are very unstable, we used the method described in [3] to find a solution.

The particle-in-a-box method is very sensitive to the approximation used for the electric field operating on the particle at coordinates x_k, between nodes of the spatial grid at which the potential is determined from the numerical solution of the Poisson equation. Unfortunately, there is no general criterion for the stability of this method.

In the finite-difference approximation, we actually approximate the force by an expression

$$E + \alpha_1 \frac{\partial E}{\partial x} + \alpha_2 \frac{\partial^2 E}{\partial x^2} + \ldots.$$

In this manner we therefore study the effect of the decrement found from the dispersion relation

$$1 = \frac{T}{m_i} \frac{(1+i\alpha_1 - \alpha_2 - i\alpha_3 + \ldots)}{1+k^2D^2} \int_{-\infty}^{\infty} \frac{\partial f/\partial v}{v+\omega/k} dv$$

on the growth of small-scale oscillations associated with the error in the calculation. For problems involving the destruction of large-amplitude ion-acoustic waves, it has been found that a method in which the electric field at point x_k is defined as

$$E_k = h^{-1}[(\varphi_{j+2} - 2\varphi_{j+1} + \varphi_j)\mu/h + \varphi_{j+1} - \varphi_j]$$
$$\mu = x_k - x_j, \qquad x_j \leqslant x_k \leqslant x_{j+1}$$

is stable.

This method has been used to solve several problems, including that of the propagation of ion-acoustic waves of supercritical amplitude (with a Mach number M > 1.6) in a nonisothermal plasma [3].

As was shown above, the initial stage in the propagation of this wave is of an oscillatory nature. The wave front accelerates, in pulses at a frequency on the order of ω_{pi}, ions accumulating in the wave system. As a result, multiple-current motion is established ahead of the shock front, and a slightly turbulent ion distribution is established behind the front, with a characteristic "arch" structure in x, v space. The subsequent behavior of such a wave is of considerable interest. Figure 1 shows the temporal evolution of the potential. Here the time t is in units of ω_{pi}^{-1}, while ξ is in units of the Debye length. The rate of change of the perturbation, specified to be initially a simple wave having an amplitude six times the density of the unperturbed plasma, then increases. During the interval t = 4-14 we find ion reflection. As the potential decreases to a value below critical ($\varphi < 1.3$), a "soliton" separates from the front with an amplitude and velocity near but slightly below the critical values.

Intense oscillations in the particle velocity are established during the particle acceleration behind the front; in turn, these oscillations cause oscillations in the density and potential. The oscillations reach values allowing trapping of ions in potential wells. Figure 2 shows part of the phase plane near the wave front at t = 25. The soliton and the ions trapped behind it are clearly visible. In the solution of this problem we used approximately 2000 particles, and the range over which the wave propagated was equal to 120 Debye lengths. Solution of this problem required 100 min on a BÉSM-6 computer.

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